Numerical Anamorphosis: an Artistic Exploration

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Anamorphoses have been know for centuries, as distorted images needing to be seen in a mirror from a special point of view in order to see the non-distorted image. During Renaissance, they have been associated to mathematical techniques for drawing such pictures efficiently, on specific mirror shapes (in the case of cylindrical or conical mirrors). We can expect in the next years a strong interest in such type of images, because of the emergence of various contexts and physical supports for image visualization (soft or deformable screens, lightmapping, projection of images on dynamic objects, etc...). Solving the numerical problem of anamorphosis in the general case belongs to the same class of problems as when the trend is to control image deformation as long as image is seen projected on, or reflected by, a non-planar surface, which can be of arbitrary shape. In this work, we show how raycasting technique, well-known in the computer graphics community, can be used to provide an efficient general framework for such rendering. We describe an effective procedure for building general anamorphoses. A generalization of the method leads to the conception of three-dimensional anamorphic sculptures, usable for 3D printing anamorphic objects. We exhibit, through several artworks, tangible and virtual examples.

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1. INTRODUCTION

The computer graphics community developed efficient tools for creating virtual images on standard flat screens. Nowadays, these tools have to face several challenges: development of deformable or curved screens, conceived to be seen from several points of view, projection of images on non-uniform surfaces (light mapping, spherical screens, peripheral illusions [14][15]), holographic projections. A simplified definition of human visual perception of an object, from the point of view of computer graphics specialists, can be thought as the projection of a texture on a geometry, seen from a certain point of view. In the simplest case, the geometry is a flat screen, and the position of the observer is also the position of the camera. Since others kinds of screens, and other ways to look at a scene become possible, how can we control the right perception of the visual message? Two sub-problems can be studied in a preliminary investigation :

—Consider a non-flat geometry: how to compute the image to be seen on this screen? The computation of distorted images will be the subject of section 4.

—Consider a visual message we want to share: what geometries can we define, that will convey this information? The definition and computation of distorted three-dimensional objects will be studied in sections 5.1 and 5.2.

The same questions arose at the time of Renaissance, when artists wanted to know how to paint three-dimensional scenes on flat canvasses, and discovered the laws of perspective. Some of them were tempted to exploit those laws to the limit, by designing either *trompe-l'œil* or anamorphoses : distorted images which need to be seen from a particular point of view, or through a specific device (pinhole, mirror) to reveal their real proportions. Anamorphoses using mirrors are called catoptric anamorphoses.

History A complete history of anamorphosis can be found in Jurgis Baltrušaitis' *Anamorphoses ou Thaumaturgus opticus* [6]. Anamorphoses begin to appear during the 16th century. First examples are credited to Leonardo Da Vinci [18]. Jean-François Nicéron (1613-1646), a Minim friar, wrote the first book about anamorphoses [7]. Among the contemporary artists working on anamorphoses, one can cite Julian Beever [3], a specialist in chalk painted anamorphoses on street pavement, and István Orosz [1], whose work explores all the artistic possibilities of catoptric anamorphosis.

Solving an anomorphosis can be defined as finding the distorted image, assuming known the shape of the mirror, the original image, and the observer's point of view. Solutions can be found either analytically, approximatively or empirically. Jean-François Nicéron described analytical and approximate methods for planar anamorphoses, cylindrical, conical and pyramidal catoptric anamorphoses. Hunt, Nickel and Gigault [11] gave a modern analytical approach of planar, conical and cylindrical anamorphoses, deriving the inverse transform in the case of cylindrical anamorphosis.

Few has been done for other kinds of mirrors :

—Andrew Crompton used an upside-down cone for his artwork at Manchester's Museum of Science and Industry. Istvan Orosz[1] is also using conical mirrors.

—Stella Battaglia and Gianni Miglietta [16] use spherical or diedral mirrors, surrounded by a distorted three-dimensional *sculpture*.

—In 1984, Fujio Watanabe [19] exhibited two artworks, where a central conical mirror was surrounded by a plastic carved cylinder. The image of this cylinder, when viewed through the mirror, revealed a human face and a skull.

Andrew Crompton introduced a new kind of mirror, Fujio Watanabe a new kind of support of distortion, while Stella Battaglia and Gianni Miglietta innovated with both new mirror shape and new support of distortion.

Strangely, those unusual versions of anamorphosis are very sparse, yet not too difficult to imagine, analyze or approximate. In

the following, we define a method for generalized anamorphoses, based on a simple principle which covers all imaginable cases of catoptric anamorphoses. The rest of the paper is organized as follows: section 3 exposes the basic principle we use to define an anamorphosis. Section 4 illustrates several concrete applications of the method. A step beyond is done in section 5.1, where we extend the method to generate three-dimensional anamorphic distorted wireframes. Section 5.2 shows that we can go still further, replacing wireframes with solid distorted objects. Finally, in section 6, we explore some possible enhancements.

2. RELATED WORK

The question of anamorphosis is, in the general case, to be associated with a physically situated computer graphics display, i.e. anamorphosis is an image that needs to be both projected (either directly or by reflexion) on a specific shape, and seen from a specific point of view. We do consider that these may be considered as being in the same intellectual spirit as of [13; 12] in the sense that anamorphosis can help making image perception independent from the shape on which it is projected. Indeed images nowadays tend to be displayed on more and more various shapes, let us mention spherical screens [5], deformable screens [20], or even whole walls [15] or rooms [14].

A large part of computer graphics is devoted to creating aesthetic images, and whole wide subjects are devoted to artistic creation, e.g. non-photo realistic rendering. Among the recent, interesting and creative works about using computer graphic techniques for art, let us mention [17], that defines a technique for 3D model volumic sculpting, that can match a constrained shadow.

3. FROM GENERIC RAYCASTING RENDERING TO NUMERICAL ANAMORPHOSIS

Notation The *surface of distortion* is the mathematical locus, often a plane, where the distorted image is drawn.

The *catoptric anamorphosis installation* is the union of the mirror, the point of view, and the surface of distortion.

The general method we describe can handle all the cases of catoptric anamorphoses discussed so far. Moreover, we will show that it can be used to define new types of catoptric anamorphoses, which in turn will lead to concrete achievements.

This method is very close to the *mechanical* technique Denis Diderot described in his Encyclopædia [9] [1]:

—Drill holes at interesting places of the design you want to transform.

—Put your mirror behind the drilled design, and hold a candle or a light (at the place where your eye should be).

—Report carefully the position where the light rays, coming out of those holes, hit the plane or the curved surface, as they will show the corresponding points of the distorted image, in such a way that one can complete the transformation.

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[1] *"On peut aussi faire méchaniquement une anamorphose de la maniere suivante : on percera de part en part le prototype à coups d'aiguille dans son contour, & dans plusieurs autres points ; ensuite on l'exposera à la lumière d'une bougie ou d'une lampe, & on marquera bien exactement les endroits, où tombent sur un plan, ou sur une surface courbe, les rayons qui passent à travers ces petits trous ; car ils donneront les points correspondans de l'image difforme, par le moyen desquels on peut achever la déformation."*

This method should work for any kind of mirror and surface of distortion, but there is no evidence that it has been used in practice, and doubts can be emitted concerning the sufficient strength of a candle light. Moreover, applying this method would be very tedious. So tedious that we can think a computer could do the job much more efficiently, using an apparented technique.
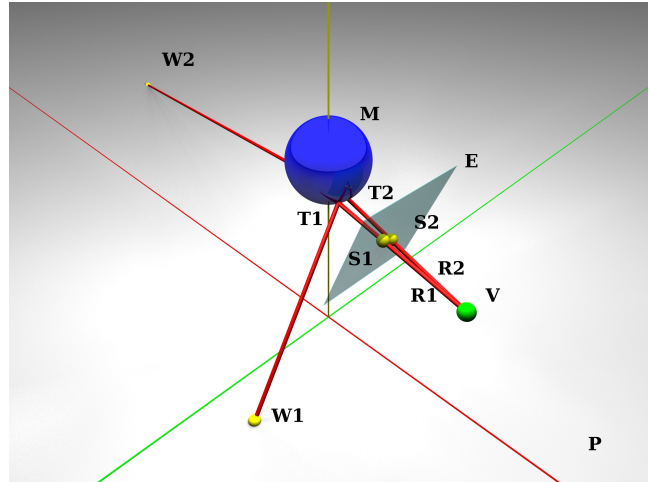

Fig. 1. The Catoptric Anamorphosis Principle

Figure 1 illustrates the principle behind our method. The conceptor of the anamorphosis installation wants the observer, placed at position V, to see a correct image into a mirror M. This image is obtained by reflection of a distorted image laying on a surface P. The image can be thought as laying on a virtual screen E. Let S_1 be a pixel from the image on E: when the observer looks at S_1, ray R_1, passing through V and S_1, hits the mirror M at point T_1. This ray is reflected and hits the plane at point W_1. So, the color of W_1 must be the color of S_1.

The reader certainly noticed that we are talking about standard ray-tracing methods, with the addition of screen E, considering the inverse path of light, and defining the color of W_1 from the desired pixel color at point S_1. Hence, standard ray-tracing softwares can be used to compute the distorted images, for any mirror shape, and any surface of distortion [2].

There are two ways for using the Catoptric Anamorphosis Principle:

—By considering each pixel of the desired image and compute the color to put on the surface of distortion. Since points that are close in the original design can lie far from each other on the surface of distortion (see for example points W_1 and W_2 in figure 1), this way of applying the method can create blank regions or other artefacts in the distorted image.

—Compute the image of a regular grid drawn on the original image. That will result in a set of quadrilaterals: use interpolation to compute the image of each elementary rectangle into the corresponding equilateral.

This last method will be the basis of our technique.

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[2] We used Povray[4]

4. ANAMORPHIC IMAGES

We are interested, in this section, in the case where input image is used only for calculating texture, on a geometry that is pre-specified. We illustrate how we applied the Catoptric Anamorphic Principle for various classic geometries. For all the examples considered here, the method will be the same:

—Set the installation: define a scene with a mirror, an image, a surface of distortion and a point of view.
—Put a grid on the image.
—Compute the image of the grid, which defines a set of couples (elementary square, corresponding distorted quadrilateral).
—Draw the distorted image of the content of each elementary square into the corresponding equilateral.
—Texture the surface of distortion with the union of all these computed images.

4.1   Example 1: anamorphic texturing of a plane
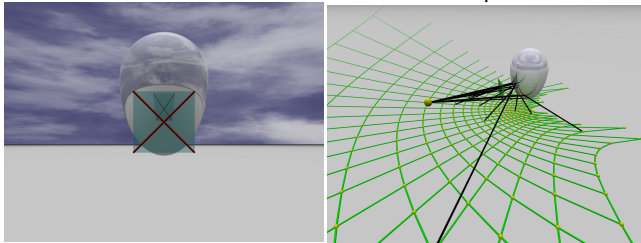

Fig. 2.   The scene.   Fig. 3.   Reflected rays.
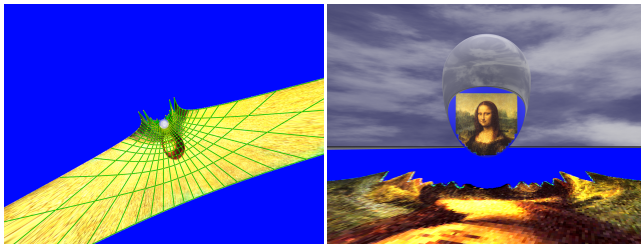

Fig. 4.   The installation from above.   Fig. 5.   Complete anamorphosis.

The simplest setting defines a horizontal plane as the surface of distortion. Figures 2 to 5 depict the complete process: figure 2 shows the catoptric installation from the observer's point of view. To illustrate the generality of the method, we use an egg-shaped mirror M. Between the mirror and the observer stands the screen E. We can see that the screen E covers the reflection of the plane into the mirror: this tells us that the whole image, excepted the two lower corners, will be reflected by the mirror. Figure 3 illustrates the Catoptric Anamorphosis Principle: rays emanating from the observer's eye go through the screen, hit the mirror and generate a distorted grid on the plane. Figure 4 shows the installation as seen from above: the distorted image is mapped on the plane. The grid has been left for illustration. Finally, figure 5 represents the completed catoptric installation, as seen from the unique point of view from where the correct image can be seen.
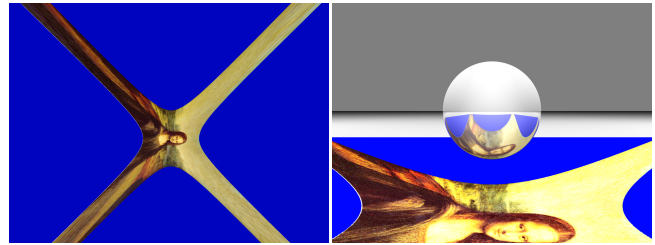

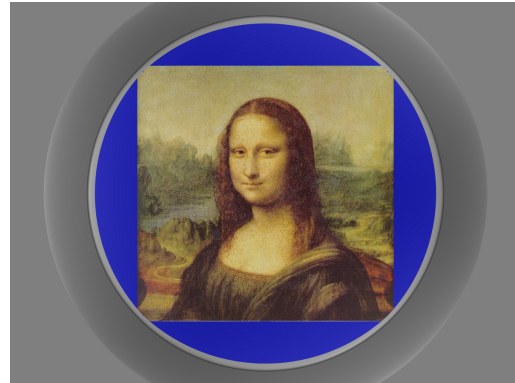Fig. 6.   Spherical mirror: distortion   Fig. 7.   Side view.

Fig. 8.   The anamorphosis as seen from beneath the sphere

Figures 6 to 8 illustrate another installation, where a spherical mirror is placed right above the head of the observer and the surface of distortion is a horizontal plane. We can notice that the distorted image goes virtually to infinity, but faraway points are of little influence in the final reflected image, and can be ignored.

The method shown in this section covers classical and historical catoptric anamorphoses, using conical or cylindrical mirrors. It can handle also Andrew Crompton or Istvan Orosz inverted conical mirror anamorphoses, and permits to investigate the case of spherical mirrors, which has not been studied widely since now. But we can go further, and explore new kinds of anamorphic installations.

4.2   Example 2: anamorphic texturing of the inner face of a cylinder

This section describes an application of the Anamorphic Principle to a new setting, only used, as far as we know, by Fujio Watanabe [19] and in a toy exhibited in London Museum of Childhood, both using a conical mirror. Since the cylinder is a developable surface, we can consider the cylinder surface as a plane, and apply the method described in section 4.1. Consider the following catoptric installation (figure 9):

The mirror is at the bottom of cylinder, the observer is right above the cylinder and looks at the mirror, the surface of distortion is the inner face of the cylinder. When unrolled, the cylinder of height l and radius r becomes a rectangle of width $2\pi r$ and height l. Computation of quadrilaterals on this plane and color interpolation are straightforward. If the observer is set at infinity, analytic solutions are easy to derive, and we can produce exact distorted images, without the interpolation step (figures 13 and 17 are exact distortions).

Figures 10 to 12 illustrate the conception of a real anamorphosis based on our method, using a Christmas ball and a plastic film
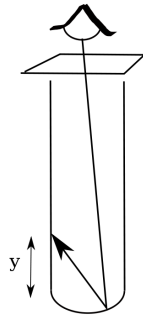
Fig. 9. The inner cylinder catoptric installation

(printer transparency). For most mirrors shapes, there will be a hole in the center of the mirror, corresponding to reflected rays which will never hit the cylinder, whatever the cylinder height. In many cases, the observer will see his eye at this place. This fact might be of interest for artists trying to involve the beholder inside their artwork.
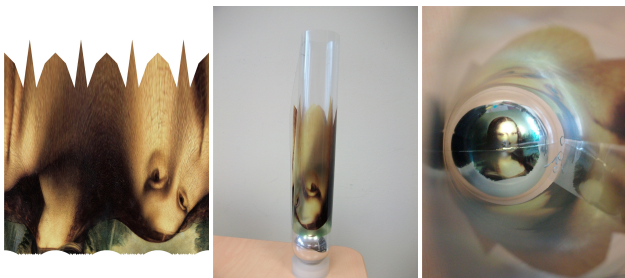

Fig. 10. The planar distorted image.

Fig. 11. The distorted image wrapped around the mirror

Fig. 12. Complete anamorphosis

Figure 13 is the distorted pattern obtained when one uses a paraboloïdal mirror inside a cylinder, and figure 14 shows the corresponding anamorphosis. This last picture is a virtual render: paraboloïdal mirrors are hard to find.

Fig. 13. Distorted pattern for a paraboloïdal mirror

Fig. 14. Paraboloïd inside a cylinder anamorphosis (virtual)

Figures 15 and 16 show the application of the process to a conical mirror inside a cylinder. In this case, the whole image is reflected in the cone. Note also how the cone is higher than the cylinder. Figure

16 is a render; the concrete installation requires very good manual skills. Figure 17 is interesting in that it shows that the distortion is not trivial: it becomes difficult to guess the final aspect of the image from this transformation. It is also the case for the distorted image of figures 10 and 13. Combining unusual mirrors and complex surfaces of distortion can make the distorted image difficult if not impossible to guess without the help of the mirror. Moreover, these distorted images can become independent aesthetical objects.
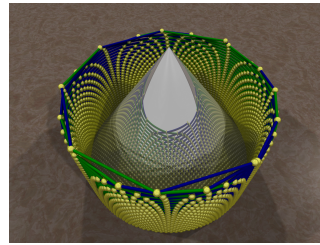

Fig. 15. Cylindrical distorted grid associated with a conical mirror.

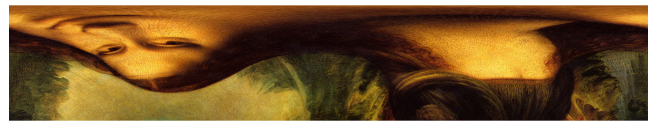Fig. 16. Cone inside a cylinder anamorphosis (virtual)

Fig. 17. The distorted pattern to apply on the cylinder.

So far we have demonstrated that an effective method exists for designing anamorphic two-dimensional distorted images, which can be revealed by a specific mirror. These images can be drawn on planes, cylinders, or any other surface, by means of surface panelling and interpolation. After the publication of this work[8], James Hopkins[10], a British sculptor, contacted us and ask whether it would be possible to define three-dimensional anamorphic objects. That is to say three-dimensional distorted objects where an image, or an object, is hidden, and that needs a mirror to reveal this hidden meaning. We call these objects *Tangible Anamorphic Objects*.

5. TANGIBLE ANAMORPHIC OBJECTS

We are interested, in this section, on how to evaluate geometry, from an input image. Few works have been done so far in that direction; let us mention artworks by Stella Battaglia sculptures[16] and Jonty Hurwitz work[2]. To our knowledge, no existing framework exists for such work. For the description of the method we propose, we focus on two possible definitions of the tangible objects we want to build: either wireframe structures made of generalized cylinders, or distorted solids, obtained as extrusion of distorted surfaces. The first objects are obtained when we consider the hidden object as a drawing made of line segments. The second kind of anamorphic objects arose when we define the hidden image as a union of elementary surfaces (triangles in our case). Both definitions were fruitful, and led to different achievements whom aesthetical appeal justify this artistic search for new kind of anamorphoses. We designed the output of our programs to make it easy to 3D-print the tangible anamorphic objects we define. 3D-printing is the ideal tool for this kind of work: the technique is so close from the mathematical definition of the objects we want to make tangible, that no

special artistic or handcraft gifts are necessary. On the other hand, rapid prototyping let us quickly validate, correct and enhance the technique. But other means of production can be considered, in co-operation with artists.

5.1 Anamorphic wireframes

5.1.1 *Adapting the method.* The idea is to replace the surface of distortion with a set of N surfaces of distortion $\{S_0, \cdots, S_{N-1}\}$. The image to be distorted is divided into line segments (a 'wired' design). Each segment is itself divided into $N-1$ sub-segments, leading to a set of N points $\{P_0, \cdots, P_{N-1}\}$ corresponding to the extremities of these $N-1$ sub-segments. We compute the image P'_i of point P_i on the surface of distortion S_i. When seen into the mirror, from the right point of view, the curve connecting the P'_i in order is an approximation of the original segment. The approximation works better as N increases. Figures 18 and 19 give an illustration of the process. Five concentric spheres are wrapped around the spherical mirror: all together they define the set of surfaces of distortion. The reflection of the curve joining the P'_i is a vertical line segment. For a better result, one must use more than five surfaces of distortion (50 surfaces are common).
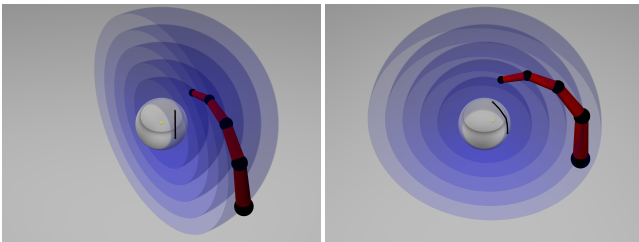

Fig. 18.   3D anamorphosis method : the reflection of the 3D curve.

Fig. 19.   The same scene seen from the line segment in the mirror is the another point of view: the reflected line is no more rectilinear.

Another point to notice: the farther the cylinders are from the mirror, the thicker they must be, in order to be perceived with the same thickness when seen by reflection. Hence, the curve connecting the P'_i is not a union of cylinders, but more exactly a set of truncated cones, or generalized cylinders.

5.1.2 *Applying the adapted method.* The method described above defines a set of tubes which we can make real by 3D printing. We made several experiments, as shown in figures 20, 21 and 23. Figures 20 and 21 correspond to the same anamorphic reflected image, with different definitions for the surfaces of distortion. In the first case, the surfaces of distortion are a set of concentric spheres with increasing radius. In the second case, the first half of the surfaces of distortion are concentric spheres of increasing radius, while the second half are the same spheres, in reverse order.

In the first case, each line segment from the original image corresponds to a curved tube beginning close to the sphere and ending far from it, while in the second case, that same line segment corresponds to an arc beginning and ending on the same surface of distortion (the surface of distortion which is the closest to the mirror).

Figure 22 is a mix between anamorphic sculpture and reflection of a cylindrical image, as described in section 4.2. Its reflection into a spherical mirror is shown in figure 23.
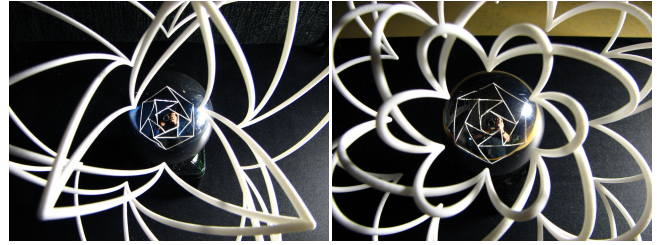

Fig. 20.   Anamorphic wireframe version 1

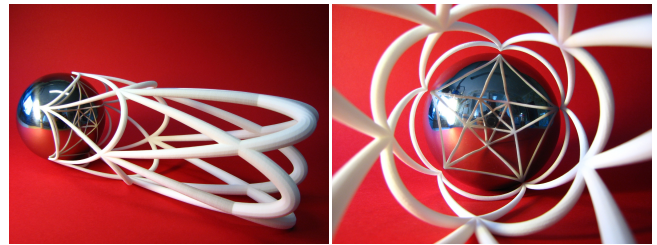Fig. 21.   Anamorphic wireframe version 2

Fig. 22.   3D-printed anamorphic wireframe sculpture on a cylinder.

Fig. 23.   Reflection of the sculpture into the mirror.

This method has its limits: one has to defined the hidden image as a set of line segments, when the original method was able to handle standard colored pictures.

5.2 Anamorphic solids

We consider here the case where the object is topologically consistant with a closed shape (i.e. wireframe can only represent object frontier, and triangles needs to be involved for shape creation). Adding a third dimension to the transformed object makes it much more difficult to understand without the mirror, which increases the interest of the anamorphosis. Moreover, this distorted shape can have its own aesthetics, just like in the case of conical anamorphoses in figure 17. But there is yet another possible enhancement: defining real three-dimensional solids instead of wireframes. For the moment, our solution is to define the transformation of an elementary two-dimensional surface (more precisely a quadrilateral) into a distorted surface bended in a three-dimensional space. As usual in Computer-Aided Geometry, surfaces can be defined as sets of triangles. We divide the quadrilateral into triangles (the larger the number of triangles, the better the result). The rest of the method is similar to the method described in section 5.1:

—Consider two opposites edges of the quadrilateral, and divide both edges in n parts, defining two sets of $n+1$ points P_i for the first edge and Q_i for the other edge.

—Let P'_i (resp Q'_i) be the images of P_i (resp Q_i) using the method described in section 5.1. The curve joining the P'_i (resp Q'_i) is the image of the first (resp second) edge of the original quadrilateral.

—Define two sets of triangles in space: $\{P'_i, P'_{i+1}, Q'_i\}$ and $\{P'_{i+1}, Q'_i, Q'_{i+1}\}$, which are images of triangles $\{P_i, P_{i+1}, Q_i\}$ and $\{P_{i+1}, Q_i, Q_{i+1}\}$ which cover exactly the surface of the original quadrilateral. We can consider that the union of all the triangles in space composes the image of the quadrilateral by catoptric anamorphosis.

Figures 24 and 25 illustrate this process: the surfaces of distortion are concentric spheres (not shown). The quadrilateral is di-
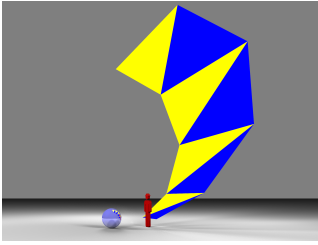
Fig. 24.   Anamorphic surface: side view.
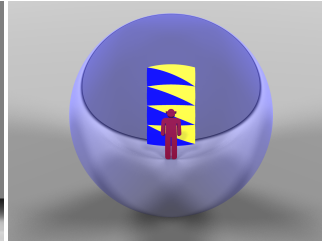
Fig. 25.   Anamorphic surface: front view.

Fig. 26.   Anamorphic sculpture (virtual): side view. (Photo M. Wilson)

Fig. 27.   Anamorphic sculpture (virtual): front view. (Photo M. Wilson)

vided into eight triangles (displayed in different colors in order to show the construction). We divide the quadrilateral along the vertical edges. For the method to be truly effective, we would have to divide also the horizontal edges: one can see that the horizontal edges are a little bit curved, instead of rectilinear. In fact, since every three-dimensional object is coded in term of elementary triangles, the method should be easier to apply on real achievements, by considering the original triangles used to define the object. If we want to make these anamorphic sculptures real, either by means of 3D-printing or CNC milling, we need to transform the computed surfaces into volumes, either by manual post-processing, or by extruding the surfaces along their normals. Both are standard methods for 3D specialists. Figures 26 and 27 are preliminary works made in collaboration with sculptor James Hopkins [10], where the computed anamorphic surfaces were extruded by a 3D modeling expert, from the information computed with our programs.

6.   CONCLUSIONS AND PERSPECTIVES

At the time when artists and scientists discovered and formalized the laws of perspective, some of them where tempted to play with these new laws, to explore their limits. Four hundred years after, new tools, new technologies allow us to go even farther in that direction. At the time of Renaissance, tools and minds were ready for defining the basic idea of anamorphoses, and particularly catoptric anamorphoses. Solutions were computed, artworks were designed for certains kinds of mirrors. We showed that nowadays, current (open-source) softwares are helpful for defining new mirrors, new surfaces of distortion, as far as planar anamorphoses are concerned. Moreover, these same tools let us explore the feasibility of more complex schemes, like for example the different kinds of anamorphic sculptures described in this paper. Lack of time prevent us for exploring other possibilities, but we have in mind several directions for further experiments, like dynamic or interactive anamorphoses, applications to human-computer interactions, data visualization.

On the other hand, if we can define technical settings for new kinds of anamorphoses, we truly need to collaborate with artists who are the only people who can find appealing and surprising applications of our programs.

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