A General Procedure for the Construction of Mirror Anamorphoses

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Abstract

Anamorphoses are distorted images that need to be seen from a special point of view in order to reconstruct the correct image (perspectival anamorphosis). In catoptric anamorphoses, a mirror is used to restore the original image. We describe an effective procedure, using ray-tracing software, to define, test and build general catoptric anamorphoses. Users can define the mirror's shape, the surface on which the distorted image lays, and set the observer's point of view.

The method can be used to virtually render and test anamorphoses designs or to output a distorted image in any suitable image format that can be printed at any scale, to obtain a real size version of the anamorphosis. We show examples with different mirrors and different distorted image surfaces.

1 Introduction

Definition and origin Artists and scientists discovered the laws of perspective at the time of Renaissance. Some of them were tempted to exploit those laws to the limit, either by designing *trompe-l'œil* or anamorphoses : distorted images which need to be seen from a particular point of view, or through a specific device (pinhole, mirror) to reveal their real proportions, and the hidden image. Anamorphoses using mirrors are called catoptric anamorphoses.

Notation The *surface of distortion* is the mathematical locus, often a plane, where the distorted image is drawn. The *catoptric anamorphosis installation* is the union of the mirror, the point of view, and the surface of distortion.

History A complete history of anamorphoses can be found in Jurgis Baltrušaitis' *Anamorphoses ou Thaumaturgus opticus* [6]. Anamorphoses begin to appear during the 16th century. First examples are credited to Leonardo Da Vinci [13]. Jean-François Nicéron (1613-1646), a Minim friar, wrote the first book about anamorphoses [8]. Among the contemporary artists working on anamorphoses, one can cite Julian Beever [3], Kurt Wenner [4], both specialists in chalk painted anamorphoses on street pavement, and István Orosz [2], whose work explores all the artistic possibilities of catoptric anamorphosis, making the distorted image as meaningful as the distorted one; his hidden portraits of William Shakespeare, Jules Verne and Edgar Allan Poe show how far the art of anamorphosis can be pushed.

We say we solve an anomorphosis when we create the distorted image, given the shape of the mirror, the original image, and the observer's point of view. Solutions can be found either analytically, approximately or empirically. Jean-François Nicéron described analytical and approximate methods for planar anamorphoses, cylindrical, conical and pyramidal catoptric anamorphoses. Hunt, Nickel and Gigault [10] gave a modern analytical approach of planar, conical and cylindrical anamorphoses, deriving the inverse transform in the case of cylindrical anamorphosis. Chantal Randour and Jean Dabbe [12] also give analytical solutions for conical and cylindrical mirrors.

Anamorphoses have been created using other mirror shapes:

- Andrew Crompton used an upside-down cone for his artwork at Manchester's Museum of Science and Industry. István Orosz [11] also used an inverted conical mirror floating above the distorted image in *Atlantis Anamorphosis II*.

- Stella Battaglia and Gianni Miglietta [7] use spherical or dihedral mirrors, surrounded by a distorted three-dimensional sculpture.
- In 1984, Fujio Watanabe [14] exhibited two artworks, where a central conical mirror was surrounded by a plastic carved cylinder. The image of this cylinder, when viewed through the mirror, revealed a human face and a skull.

In these examples, Andrew Crompton and István Orosz introduced new mirror shapes, Fujio Watanabe introduced a new method of distortion, while Stella Battaglia and Gianni Miglietta innovated with both new mirror shape and new methods of distortion.

Surprisingly, these unusual types of anamorphoses are rare, even though not too difficult to imagine, analyse or approximate.

A general method: The general method we describe can handle all the cases of catoptric anamorphoses discussed above. Moreover, we will show that it can be used to define new types of catoptric anamorphoses, which in turn could be developed into physical examples.

This method is close to the *mechanical* technique Denis Diderot described in his Encyclopædia [9]:

> Drill holes at interesting places of the design you want to transform. Put your mirror behind the drilled design, and hold a candle or a light (at the place where your eye should be). Report carefully the position where the light rays, coming out of those holes, hit the plane or the curved surface, as they will show the corresponding points of the distorted image, in such a way that one can complete the transformation

Diderot's method should work for any kind of mirror and surface of distortion, but there is no evidence that it has been used in practice, and there are concerns about the sufficient strength of a candle light. Moreover, applying this method would be very tedious. So tedious that perhaps a computer could do the job much more efficiently, using an comparable technique.

Organization: Section 2 explains the basic principle behind our method, section 3 describes how this principle is applied using existing software. Section 3 also shows some examples that use different catoptric anamorphoses and finally discusses some future work that we have not yet explored.

2 The Catoptric Anamorphosis Principle

Figure 1 illustrates the principle behind our method. The designer of the anamorphosis installation wants the observer, placed at position V, to see a correct image into a mirror M. This image is obtained by reflection of a distorted image laying on a surface P. The image can be thought of as laying on a virtual screen E. Let S_1 be a pixel from the image on E: when the observer looks at S_1, ray R_1 passes through V and S_1, hits the mirror M at point T_1. This ray is reflected and hits the plane at point W_1. Reversing the direction of R_1, we can say that W_1 is seen by the observer, who looks in the direction of the mirror, in the exact direction of S_1. This reasoning is true for each pixel. The distorted image is the union of all W_i, each associated with a pixel S_i from the original image. We remove Screen E, which was used for explanatory purposes. The position of pixel S_2 shows how much an image can be distorted: two neighbouring pixels in the original image can result in very distant points on the surface of distortion.

Of course, some rays passing through V and S_i never hit the mirror, and some other rays that hit the mirror never reach the plane P : this will result in blank regions in the mirror. One can control which part of the original image will be represented on P by moving the screen E nearer or farther, or by modifying the position of V; Figure 3 for instance, illustrates one viable alignment.

We see how similar this principle is to Diderot's idea: V plays the role of the candle, pixels S_i are the holes drilled in the design, and each W_i corresponds to a spot of light generated by the candle through a hole.

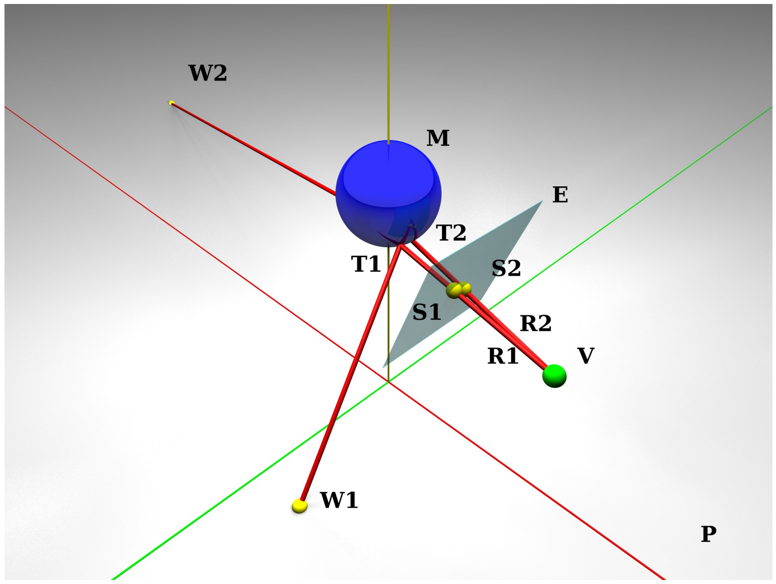

Figure 1 : *The Catoptric Anamorphosis Principle*

This general principle naturally calls to mind a method to produce the distorted image: map an image onto the virtual screen E. For each pixel S_i of this image: compute and output the coordinates of the corresponding pixel W_i, together with the color of S_i. Finally, use the collected data to produce the distorted image.
However, another classic method for constructing anamorphoses might save us from mapping every pixel through the mirror. This method is to place a grid of control points on the original design, and compute the corresponding distorted grid to use as a guide to produce the distorted image. Combining control points with Diderot's idea, we drill regularly spaced holes in the design, and join adjacent spots of light with straight lines. Ideally, this gives a network of quadrilaterals. Drawing the distorted image entails mapping pixels from each elementary square of the original design to its image in the corresponding quadrilateral.
As we will see, our technique uses this concept of a distorted control grid, rather than mapping every pixel through the mirror.

3 The Method in Practice

A ray tracer is a program that takes as input a scene description, and outputs the image of this scene. A scene description is a text file containing the description and position of objects, light sources and camera. The output image renders the objects as they are seen from the camera's position, with lights and shadows corresponding to the light sources. A ray tracer works by sending individual rays and then follows their paths through the scene. The ray tracer needs to be able to detect where a ray hits an object and to compute the reflected ray. Pov-Ray [5] is a ray tracer, that contains a programming language (with built-in geometrical vector operations). It implements a very useful function for our purpose: the *trace* function takes as input a starting point, a direction vector and an object, and outputs the position on the object where a ray (that starts from the starting point and travels in the given direction) hits the given object (if it hits it). Furthermore, *trace* returns the vector normal to the object's surface at this point which could then be used to compute the direction of the reflected ray. Using a ray tracer such as Pov-Ray, we implement our method: let V be our initial point and M the object representing the mirror's shape. Define a set of equally distant coplanar points between V and M (the *holes* of the grid in our screen E). Each of those points can be associated with two integers (a,b) denoting its position on the grid. Iteratively call the *trace* function to obtain the position where each ray coming out from V, passing through each *hole*, hits the mirror. Compute the reflected ray and call the *trace* function again to obtain the position (x,y,z) where the reflected ray hits the surface of distortion. If the ray hits the surface of distortion, output (a,b,x,y,z) to a file.

Using the programming environment of our choice, we can process the information in this file to produce a transformed image in two steps: compute the locations of the distorted grid points within the output image and then for each square in the original grid, use bilinear interpolation to transform its image into the corresponding quadrilateral in the output image.

Each time the points associated with integers $(a,b),(a,b+1),(a+1,b),(a+1,b+1)$ are all present in the output file, we assume that the entire corresponding square in control grid is reflected in the mirror. If it isn't we can detect this when we produce the final render. Each initial square is then associated with the quadrilateral in the distorted grid, provided it is reflected by the mirror on the surface of distortion.

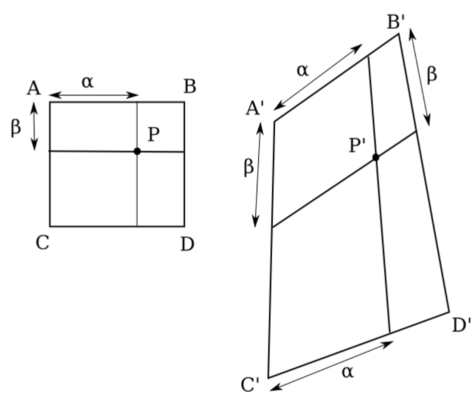

Figure 2 : *Mapping a square to a quadrilateral*

We can define the transformation that maps each square to the corresponding quadrilateral. Figure 2 illustrates the idea behind this mapping: let P a pixel inside square $ABCD$. Define α as the projection of P onto AB, and similarly β, the projection of P onto AC. Mark the sides of $A'B'C'D'$ with the same proportions of α and β on the sides of $ABCD$ to find P', the image of P. Implementing the mapping is straightforward, one just needs to scan all the square's pixels. We note that a reverse mapping from a quadrilateral in the output image to a square in the undistorted grid would be better for implementation purposes.

If the surface of distortion is a plane or a cylinder, we can use built in functionality in Pov-Ray to map the computed distorted image onto this surface, and render the scene again in order to verify the validity of the method. For more complicated surfaces of distortion, mapping might not be so easy.

3.1 Planar surfaces of distortion

Without loss of generality, we can assume that the plane is horizontal. Figures 3 to 6 depict the complete process: figure 3 shows the catoptric installation from the observer's point of view. The mirror M is an egg-shaped surface. Between the mirror and the observer stands the screen E. The surface of distortion is the horizontal plane below. We can see that the screen E covers the image of the plane in the mirror: this tells us that the whole image, except for the two lower corners, will be reflected by the mirror. Figure 4 illustrates the catoptric anamorphosis principle: rays emanating from the observer's eye go through the screen, hit the mirror and generate a distorted grid on the plane. Figure 5 shows the installation as seen from above: the distorted image is mapped on the plane. The grid has been included for illustration. Finally, figure 6 represents the completed catoptric installation, as seen from the unique point of view where the correct image can be seen.

For some catoptric installations, depending of the shape of the mirror or the point of view, the distorted grid could be very large. If we have to design a real-world anamorphosis installation, we would have to deal with such problems: it's not likely that we could use any unbounded surface! Our method can easily deal with such limitations, by considering only the quadrilaterals that fall on some predefined bounds. On the other hand, remote pixels tend to be not very significant for the reflected image. It is up to the designer to compromise between precision and appearance, and virtual rendering can help.

The methods shown in the above example are valid for classical catoptric anamorphoses, that use either conical or cylindrical mirrors. It can also handle Andrew Crompton's and István Orosz's inverted conical

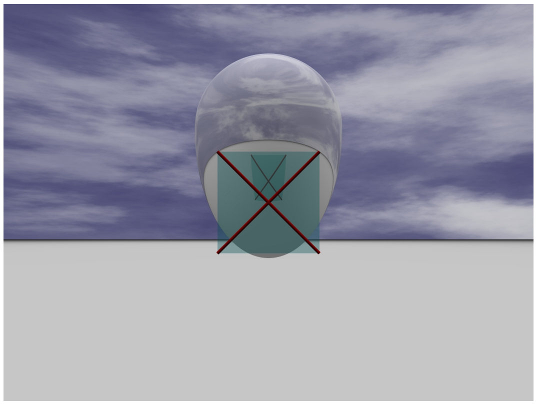

Figure 3 : *The scene*

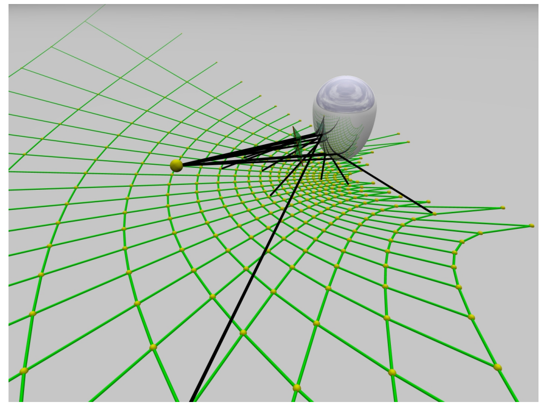

Figure 4 : *Reflected rays*

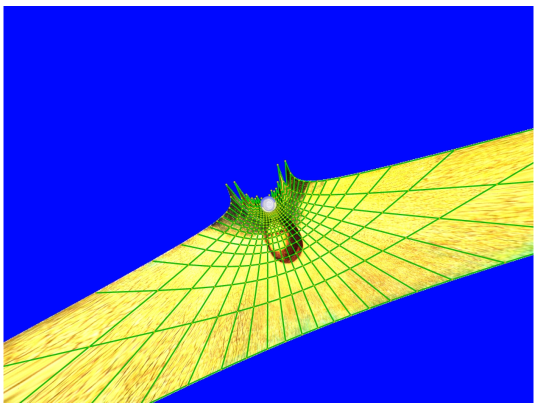

Figure 5 : *The installation seen from above*

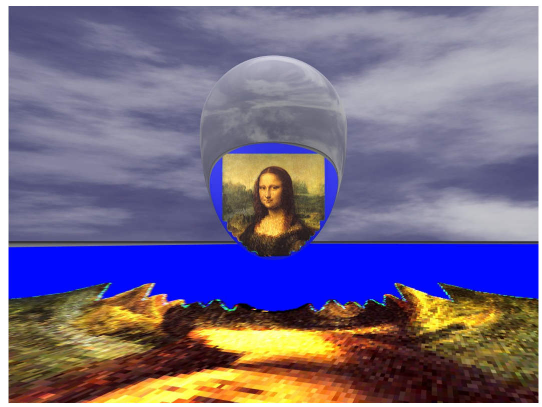

Figure 6 : *The completed anamorphosis*

mirror anamorphosis and allows investigations into anamorphoses that use spherical mirrors which have not been very common up until now. For example, an interesting setting could be easily defined with a spherical mirror standing right above the observer (figures 7 to 9).

The method gives good results when each original grid square covers a *uniform* region of the mirror, with smooth and continuous curvature. Otherwise, the square's corners can be reflected in completely different directions, the corresponding quadrilateral can not be considered as the image of the square, and interpolation becomes meaningless. Reducing the size of elementary squares results in better approximations, with an increase in computing cost, but experience shows that computing time always remains reasonable.

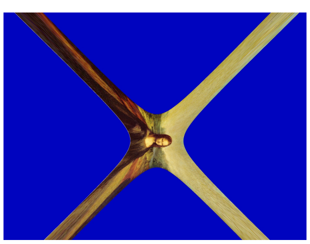

Figure 7 : *Spherical mirror: the distorted image*

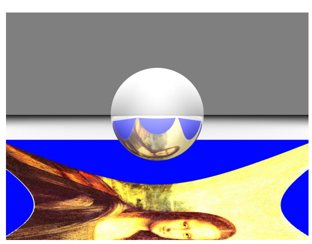

Figure 8 : *The installation seen from the side*

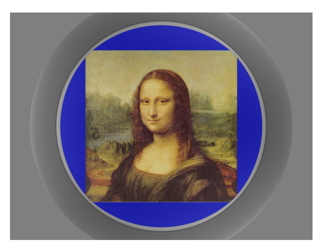

Figure 9 : *The anamorphosis as seen from beneath the sphere*

3.2 Cylindrical surfaces of distortion

Since the cylinder is a developable surface, we will compute the distorted image on the (now bounded) plane corresponding to the developed cylinder. We can then either virtually map the image onto a cylinder, using Pov-Ray, and verify the anamorphosis or print, roll and wrap it around the mirror.

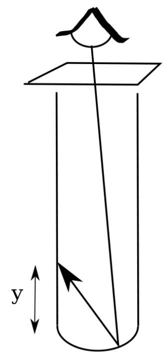

Figure 10 : *The inner cylinder catoptric installation*

Consider the following catoptric installation (figure 10): the mirror is at the bottom of a cylinder, the observer looks at the mirror, the surface of distortion is the inner face of the cylinder . When unrolled, the cylinder of height l and radius r becomes a rectangle of width $2\pi r$ and height l. Consider the data (a,b,x,y,z) output by the application of the catoptric anamorphosis principle, as defined at the beginning of section 3. Let y be the vertical coordinate: y directly give the height of the pixel on the plane, while the coordinate on the X-axis, relatively to the rectangle, is given by $2(\arctan(x/z)+\frac{\pi}{2})r$. All the information we need to first define the distorted quadrilaterals, and then interpolate the distorted image are available in the data file generated by the application of the anamorphosis principle. This process makes no assumption about the shape of the mirror, and indeed several shapes were used successfully: cones, spheres, paraboloids, both concave or convex. Because the surface of distortion is bounded, practical constructions are easy. Figures 11 to 13 illustrate the design of a real anamorphosis based on our method, using a christmas ball and a transparent for slides. For most mirror shapes, there will be a hole in the center of the mirror, corresponding to reflected rays which never hit the cylinder, whatever the height of the cylinder will be. In many cases, the observer will see his eye at this place.

Figure 11 : *The planar distorted image*

Figure 12 : *The distorted image wrapped around the mirror*

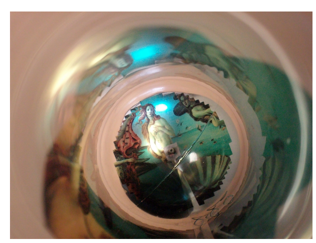

Figure 13 : *The anamorphosis*

Figures 14 and 15 show the application of the process to a conical mirror inside a cylinder. In this case, the whole image is reflected in the cone. Note also that the cone is higher than the cylinder. This example is only a prototype, but serves to show that our method can actually lead to physical examples. Anybody with a little bit more manual skills than the author's would have made something more appealing. Figure 16 is the distorted pattern obtained when one uses a paraboloidal mirror inside a cylinder, and figure 17 shows the related anamorphosis. This last picture is a virtual render, since we have not yet found nor built paraboloidal mirror.

Figure 14 : *Cone inside a cylinder installation*

Figure 15 : *Cone inside a cylinder: render gives better results*

Figure 16 : *Distorted pattern for a paraboloidal mirror inside a cylinder*

Figure 17 : *Paraboloidal mirror inside a cylinder anamorphosis*

3.3 Future work

As soon as we can describe the shape of a mirror in a format suitable for our ray tracer, we can use the method to test the catoptric installation. Experiments with simple concave mirrors inside a cylinder have also been attempted, and results are promising. Other surfaces of distortion are worth trying, among which developable surfaces play a particular role, as they can lead to planar distorted images. Polyhedral mirrors may also lead to interesting results: István Orosz and Stella Battaglia already explored this aspect. We can imagine more general surfaces of distortion. What about a curved mirror at the bottom of a cooling tower, where the surface of distortion is the inner surface of the hyperboloid, and the observer has to walk on a footbridge at mid-height and look down below?

4 Conclusion

The method we describe allows one to test, at a very low cost, anamorphosis design on general or unusual mirrors, letting the programmer judge for its aesthetical value, and then decide to construct it or not. Even if a construction is not planned, it can lead to the creation of high quality renders and animations.
The method can handle any mirror shapes, but experiments with complex concave mirrors or with mirrors where multiple reflections can occur often give very poor results: in those cases the transformed grid overlaps itself, and no "good" distorted images can be produced. Using the method to study more precisely those degenerated cases could lead to interesting results.

The method is not completely automated, and there are quite a number of places inside the source code where the user has to perform some tuning, but experience shows that a new catoptric installation can be set, tested and displayed in less than an hour. A web page containing a tutorial explaining the method step by step, together with several analytical results has been published and continues to be maintained and updated [1].

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